

Design & Optimization of a Compton Camera for Nuclear Medicine Applications

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C-SPRINT Camera

Silicon detector:

$3 \times 3 \times 0.1 \text{ cm}^3$

2 keV energy resolution

1.2 mm pixels

Sodium Iodide detector ring:

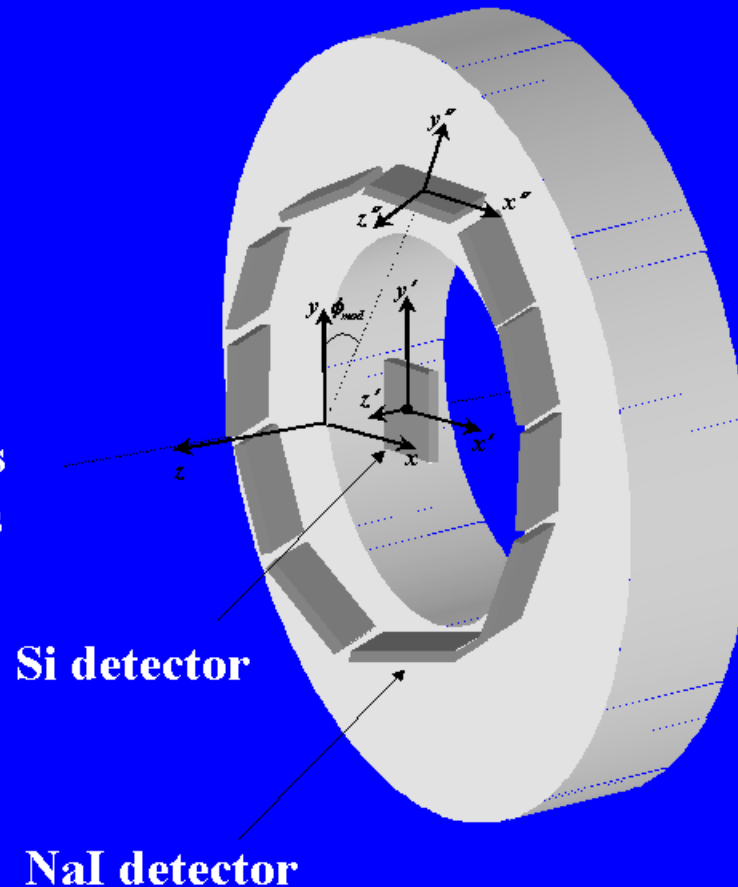
50 cm dia. 10 cm length annulus

11 modules of 1.27 cm thickness

20 photomultiplier tubes per
module

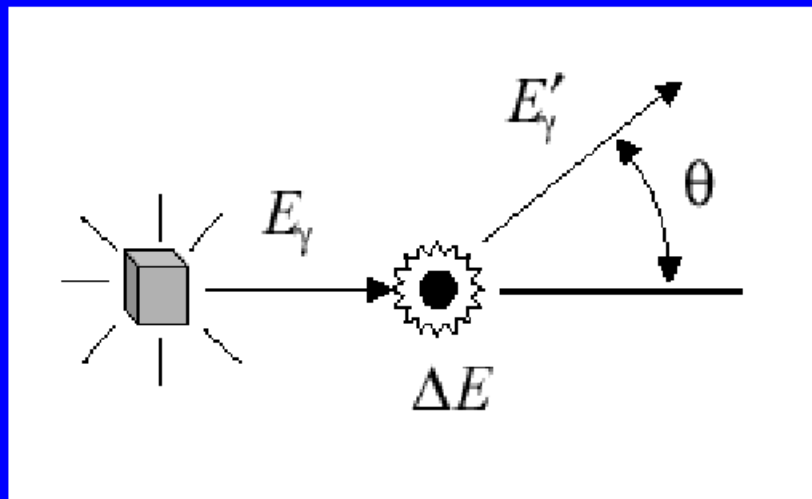
8 keV energy resolution

3 mm spatial resolution



Courtesy: Prof. W.L.Rogers, UMich

Compton Scattering



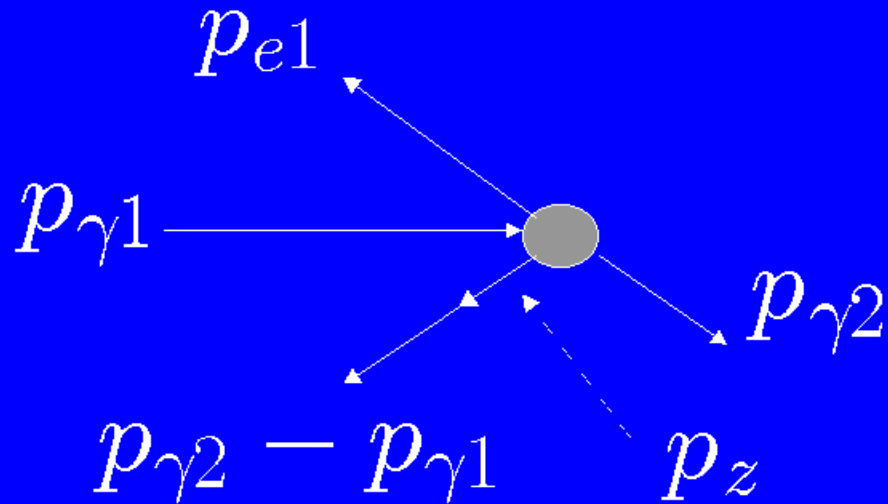
$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{mc^2} (1 - \cos \theta)}$$

E_γ : Initial energy of photon

θ : Scattering angle

E'_γ : Energy of scattered photon

Doppler Broadening



$p_{\gamma 1}$: Initial gamma-ray momentum, $p_{\gamma 2}$: Final gamma-ray momentum

p_{e1} : Initial electron momentum

p_z : Projection of electron momentum on the gamma-ray
momentum transfer vector

Doppler Broadening

Free electron at rest

$$\Delta E = E_{\gamma} \left[1 - \frac{1}{1 + \frac{E_{\gamma}}{mc^2}(1 - \cos\theta)} \right]$$

ΔE : Energy lost to electron

Bound electron in motion

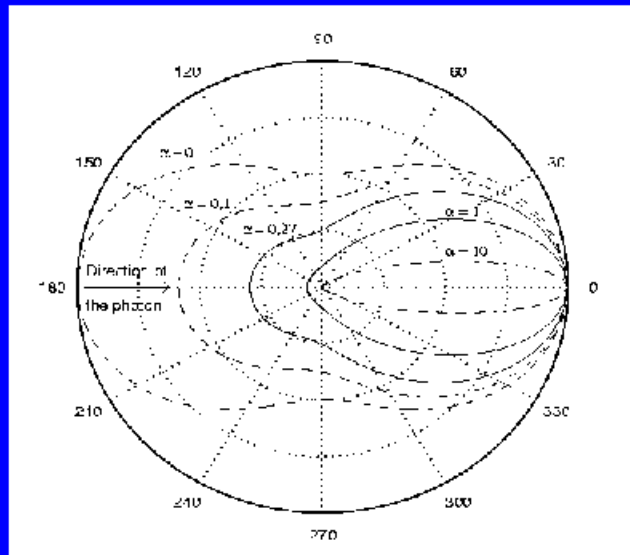
$$p_z c = - mc^2 \frac{1 - \frac{E'_{\gamma}}{E_{\gamma}} - \frac{E'_{\gamma}}{mc^2}(1 - \cos\theta)}{\sqrt{1 + \left(\frac{E'_{\gamma}}{E_{\gamma}}\right)^2 - 2\frac{E'_{\gamma}}{E_{\gamma}}\cos\theta}}$$

Klein-Nishina Equation: Free Electron

Gives the probability of a photon scattering into a given solid angle

For an *unpolarized* photon interacting with a free electron:

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{4} \left(\frac{E'_\gamma}{E_\gamma} \right)^2 \left[\frac{E'_\gamma}{E_\gamma} + \frac{E_\gamma}{E'_\gamma} - 2\sin^2\theta \right]$$



r_0 : Classical electron radius

Ω : Solid angle the photon scatters into

σ : Cross-section

Klein-Nishina Equation: Bound Electrons

For an *unpolarized* photon interacting with a bound electron:

$$\frac{d^2\sigma}{d\theta dE'_\gamma} = \frac{\pi r_0^2 \sin\theta}{E_\gamma} \left(\frac{E'_\gamma}{E_\gamma} + \frac{E_\gamma}{E'_\gamma} - 2\sin^2\theta \right) \frac{p_z J_n(p_z)}{1 - \frac{E'_\gamma}{E_\gamma} \frac{E_\gamma}{mc^2} (1 - \cos\theta)}$$

σ_n : Cross-section of the n^{th} shell,

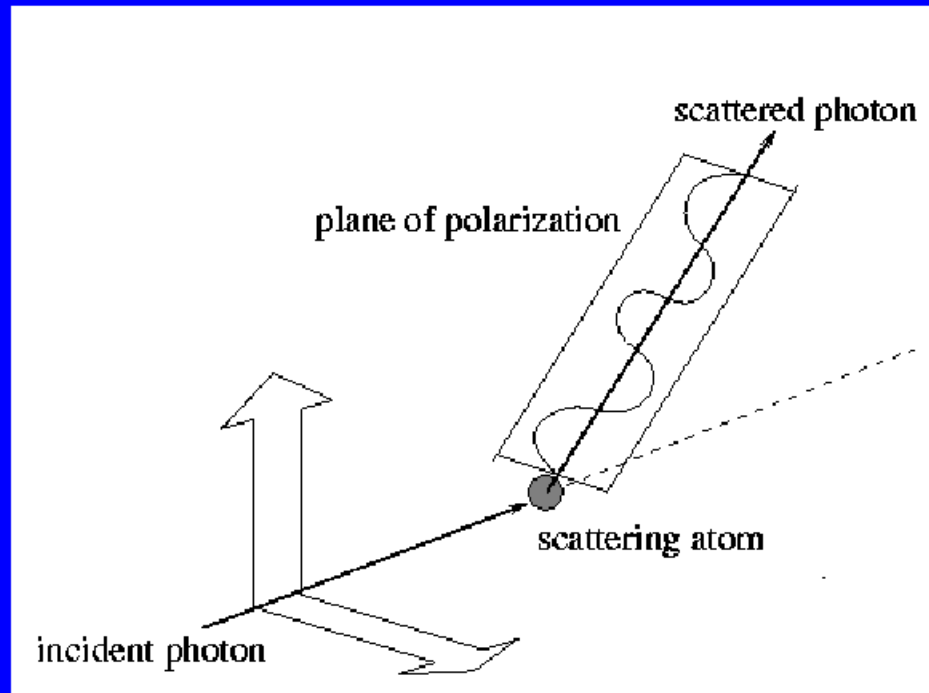
'Compton profile':
 $J_n(p_z)$: the electron momentum
distribution for the n^{th} shell

Polarization Of Photons

Photons polarize linearly on Compton scattering

Highly probable (>0.95) for energies < 511 keV

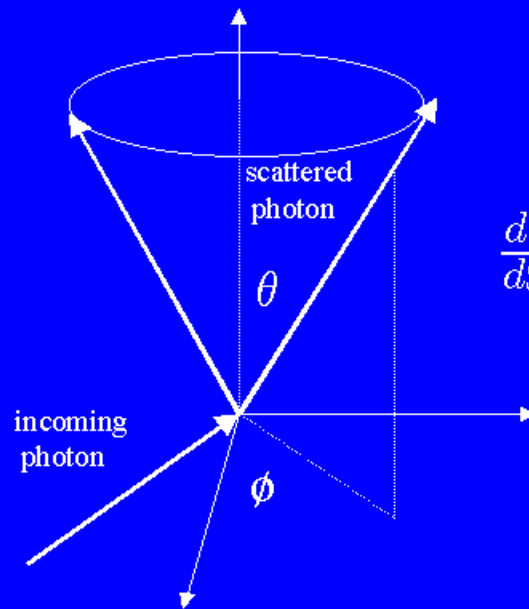
Linear polarization: Electric vector of the photon oscillates in a plane



Effect of Polarization on Scatter

Compton scattering of a linearly polarized photon is not symmetric in the azimuthal angle (ϕ) distribution

Polarized photons preferentially scatter with ϕ normal to the plane of polarization

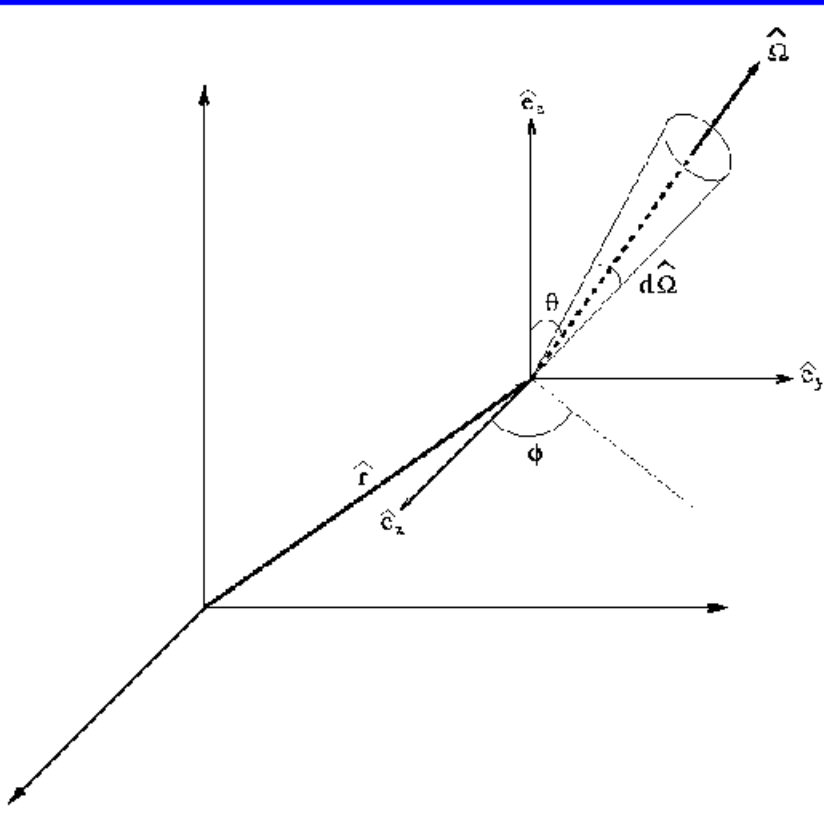


Klein-Nishina equation for polarized photons

$$\frac{d\sigma}{d\Omega} = \frac{r_0^4}{4} \left(\frac{E'_\gamma}{E_\gamma} \right)^2 \left[\frac{E_\gamma}{E'_\gamma} + \frac{E'_\gamma}{E_\gamma} - 2 \sin^2 \theta \cos^2 \phi \right]$$

The Boltzmann Photon Transport Equation

Computes the particle flux at each point in the region of interest



$$\begin{aligned} \psi(\hat{r}, \hat{\Omega}, E) = & \\ & \psi(\hat{r}, \hat{\Omega}, E)_{propagated} + \\ & \psi(\hat{r}, \hat{\Omega}, E)_{scattered} + \\ & \psi(\hat{r}, \hat{\Omega}, E)_{absorbed} + \\ & \psi(\hat{r}, \hat{\Omega}, E)_{generated} \end{aligned}$$

The Boltzmann Equation (Cont'd)

The terms in the equation:

$$\psi \left(\hat{r}, \hat{\Omega}, E \right)_{propagated} = \psi \left(\hat{r}, \hat{\Omega}, E \right) e^{\int (\hat{r}, \Omega' \rightarrow \Omega, E) d\Omega' dV'}$$

$$\begin{aligned} \psi \left(\hat{r}, \hat{\Omega}, E \right)_{scattered} &= -\sigma_s \left(\hat{r}, \hat{\Omega}, E \right) \psi \left(\hat{r}, \hat{\Omega}, E \right) dE d\hat{\Omega} dV \\ &+ \iiint \sigma_s \left(\hat{r}, \Omega' \rightarrow \Omega, E' \rightarrow E \right) \psi \left(\hat{r}, \hat{\Omega}', E' \right) dE d\hat{\Omega}' dV' \end{aligned}$$

$$\psi \left(\hat{r}, \hat{\Omega}, E \right)_{absorbed} = \sigma_{\tau} \left(\hat{r}, E \right) \psi \left(\hat{r}, \hat{\Omega}, E \right) dE d\hat{\Omega} dV$$

$$\psi \left(\hat{r}, \hat{\Omega}, E \right)_{generated} = 0$$

σ_s : Scattering cross-section

σ_{τ} : Photoelectric cross-section

Solving the Photon Transport Problem

GEANT (GEometry ANd Transport) Toolkit from CERN

EPDL Evaluated photon data library from LANL

Algorithm:

Photon is sourced with a predetermined energy

Direction and path length are randomly sampled

After transport, sample possibility and type of interaction

If photoelectric: Sample new photon

If Compton: Sample new scatter angle and path length

If required: Doppler broadening and polarization

**Continue the process until an energy cutoff (≤ 30 keV)
is reached or photon exits volume of interest**

Optimizing the Compton camera

Design Parameters:

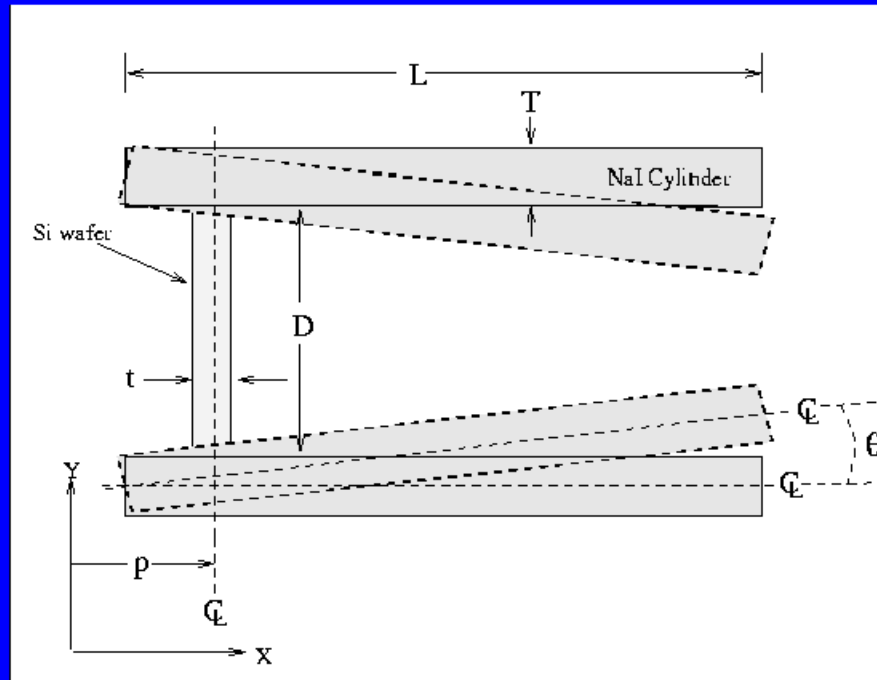
t : thickness of Si wafer

D : diameter of Si and NaI detectors

L : length of NaI cylinder

p : position of Si wafer

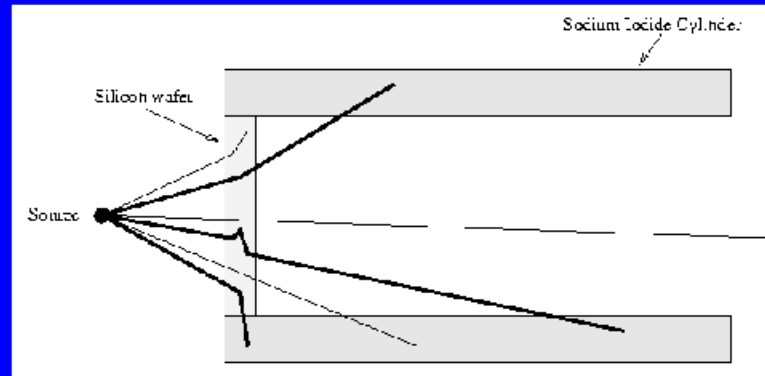
θ : angle of the NaI cone



Test the camera with two energies: 140 keV (^{99m}Tc)
511 keV (^{18}F)

Performance Measures

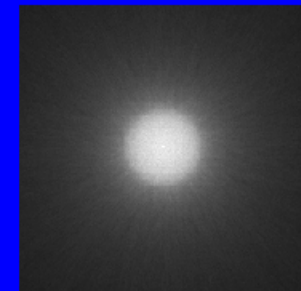
Sensitivity: Number of good counts for a preset number of photons



Resolution: Full Width at Half Max (FWHM) of reconstructed profile

Contrast: Captures both sensitivity and resolution. Task based Image discs superimposed over active backgrounds. Reconstruct and measure contrast

$$C = \frac{R_d - R_a}{R_d + R_a}$$



Technetium-99m Simulations

140 keV

Linear attenuation coefficient (μ_l) in Si = 0.337 cm⁻¹

Mean free path in Si: 2.96 cm

Linear attenuation coefficient (μ_l) in NaI = 0.59 cm⁻¹

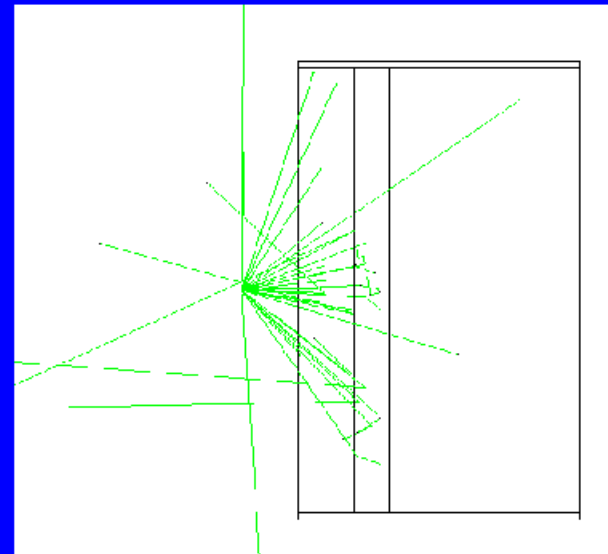
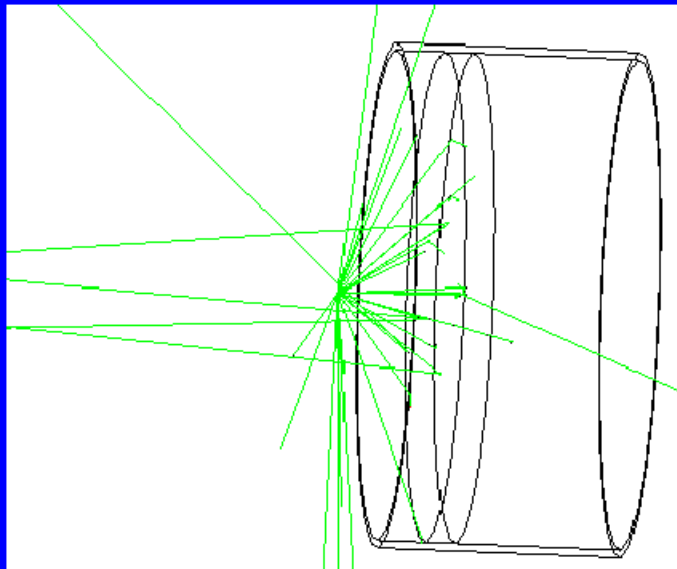
Mean free path in NaI: 2.16 cm

Choice of wafer thicknesses: 2, 3, 4, 5, 6 cm

NaI lengths: 1, 10, 20, 30, 40, 50, 60 cm

NaI & Si diameters: 1 to 125 cm at 5 cm intervals

Sample Simulation



Point source: 140 keV, Source to camera distance: 10 cm

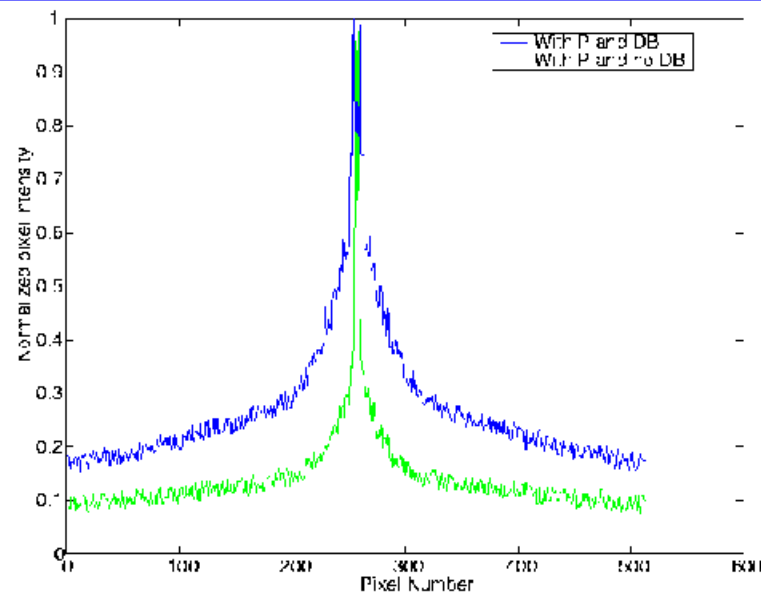
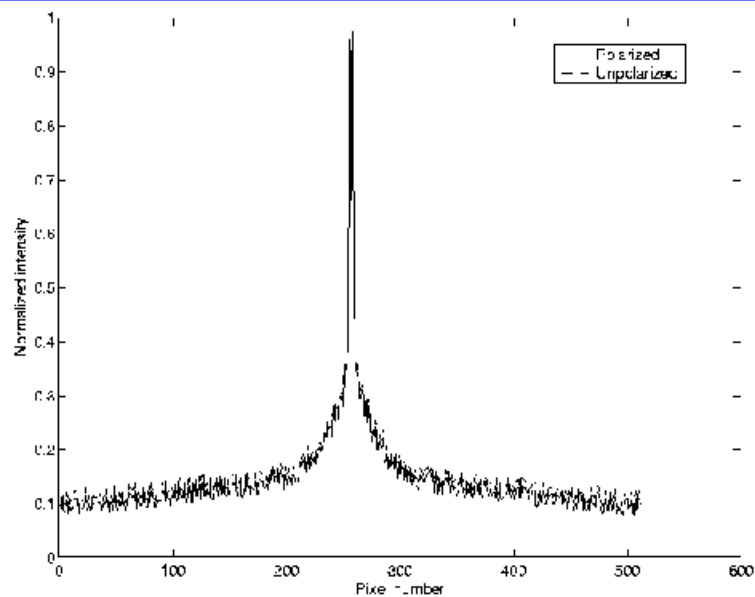
Thickness: 3cm, Diameter: 50 cm, Length: 25cm, Position: 10 cm

30 photon tracks are shown

Effects of Doppler Broadening and Polarization

Doppler broadening: Degraded both Sensitivity and Resolution

Polarization: Had no effect on Sensitivity or Resolution



Sensitivity Computations for 140 keV

Simulations were performed at various choices of t, L, and D

1 million source histories per simulation

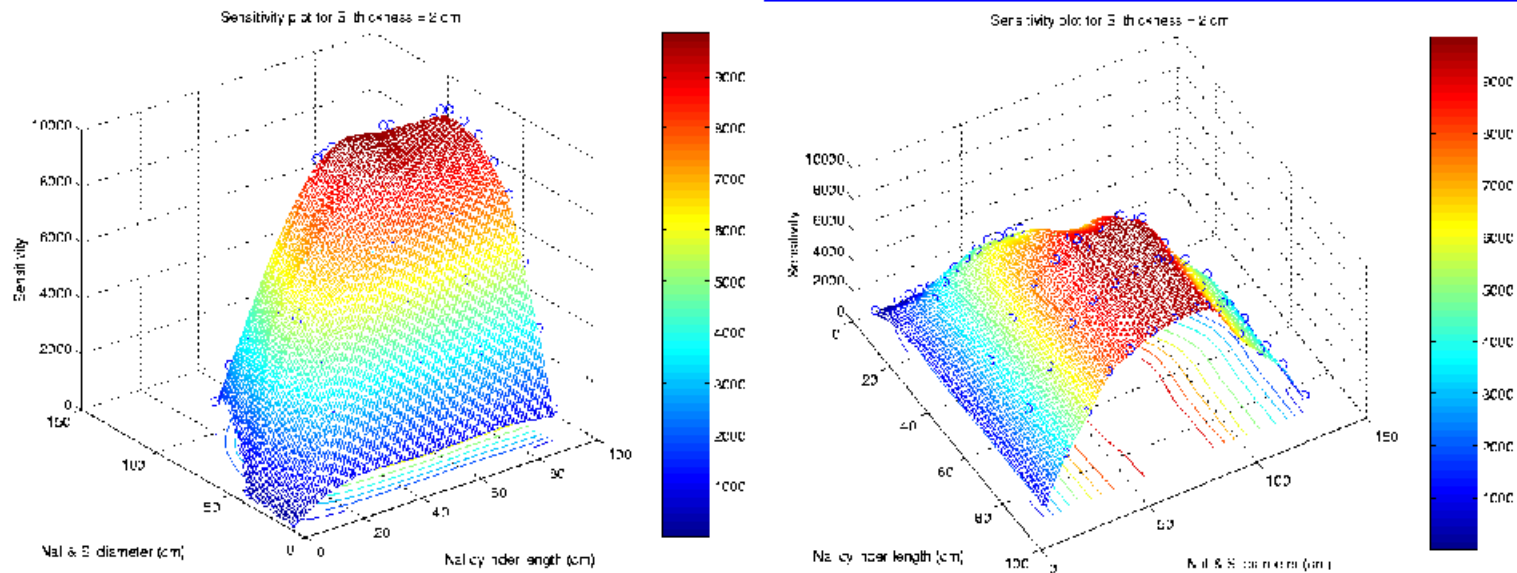
Sensitivities were computed directly from coincident events

Resolutions were computed from reconstructions

Splines were fit to the data

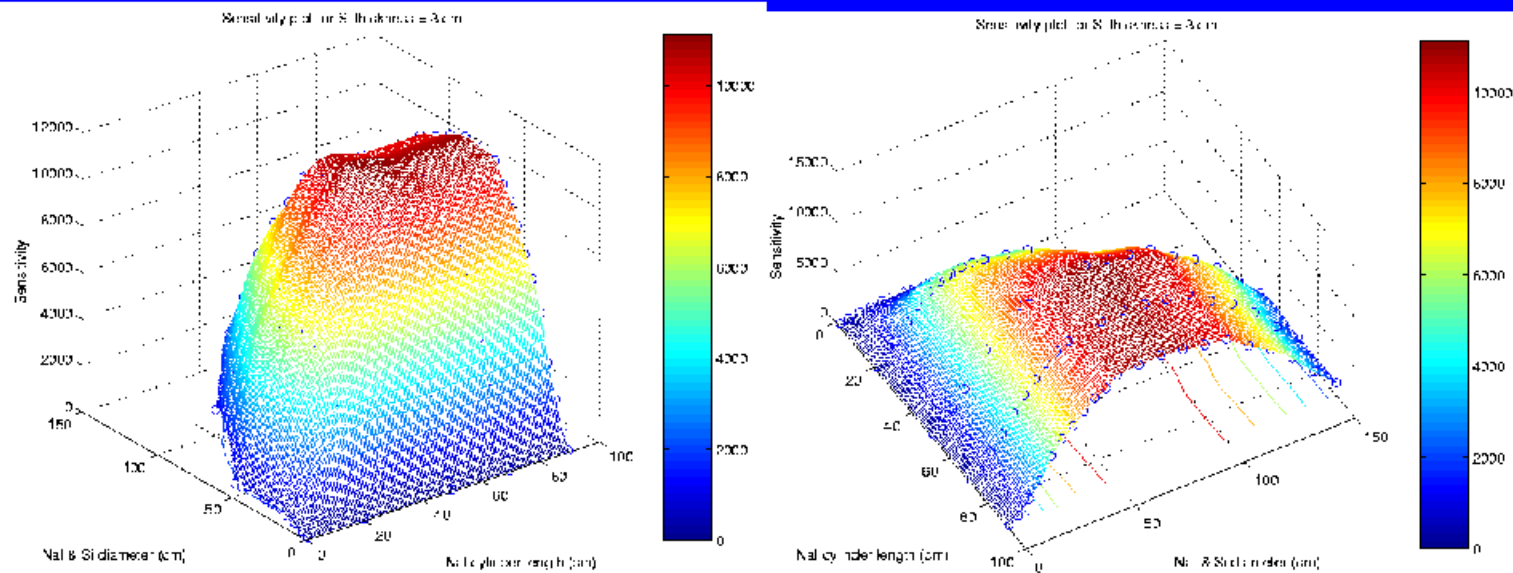
Maxima were computed from the spline fits

140 keV Simulations: Sensitivity at $t = 2$ cm



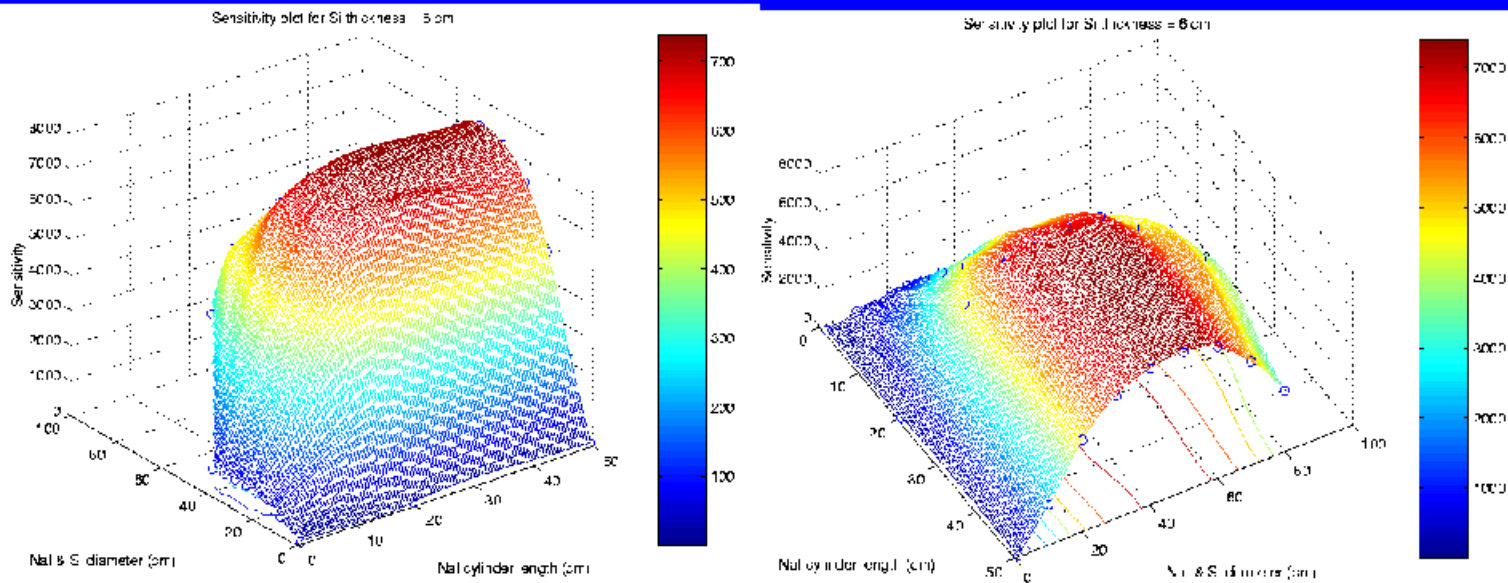
Optimum: $t = 2$ cm, $L = 52.6$ cm, $D = 83.7$ cm, Sensitivity = 11558

140 keV Simulations: Sensitivity at $t = 3$ cm



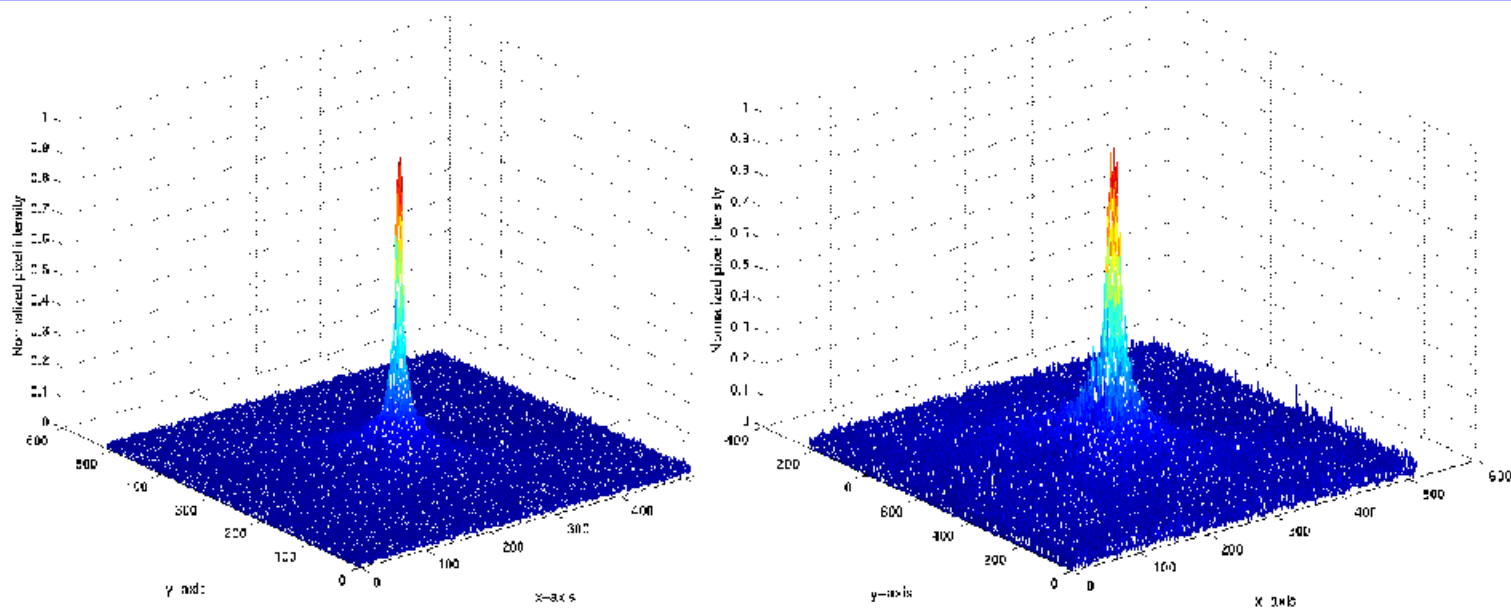
Optimum: $t = 3$ cm, $L = 51.4$ cm, $D = 72.1$ cm, Sensitivity = 11987

140 keV Simulations: Sensitivity at $t = 6$ cm



Optimum: $t = 6$ cm, $L = 47.8$ cm, $D = 52.3$ cm, Sensitivity = 7245

140 keV FWHM Resolutions



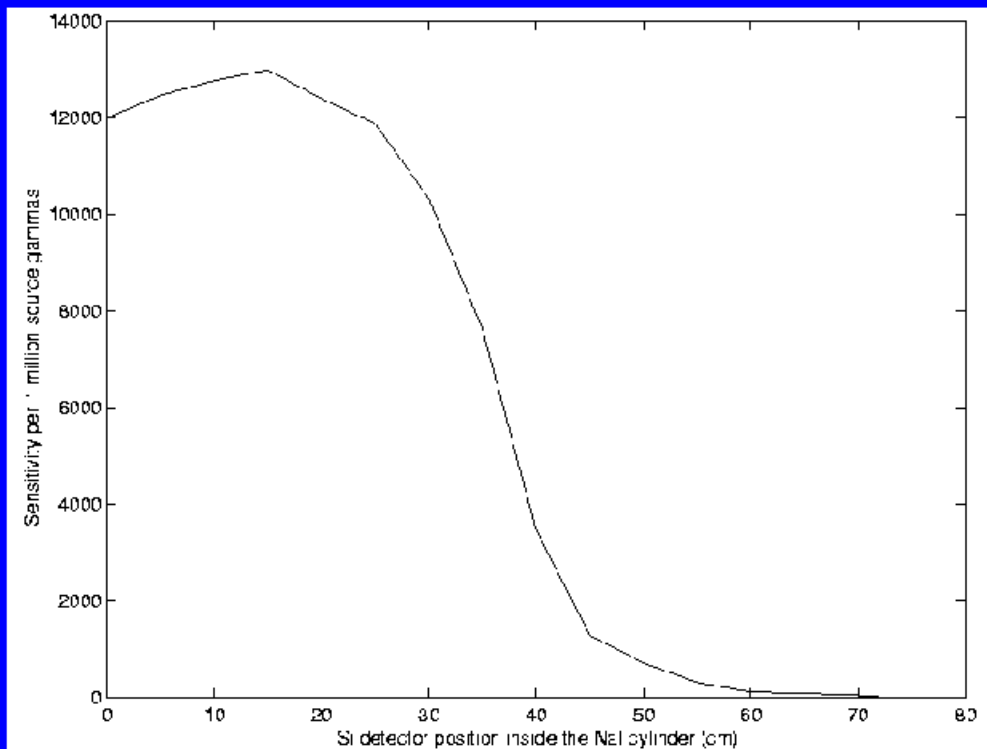
Source at 10 cm from camera

$t = 3$ cm, $L=51.4$ cm $D=72.1$ cm
Sensitivity = 11558
FWHM = 0.964 cm

$t = 4$ cm, $L=51.1$ cm, $D=60.3$ cm
Sensitivity = 9657
FWHM = 1.7 cm

Optimum Design for 140 keV Imaging

Position of the Si detector was adjusted to maximize sensitivity



Optimum setting:

t = 3 cm

L = 51.4 cm

D = 72.1 cm

angle = 16°

p = 17 cm

Sensitivity: 13451

Resolution: 0.953 cm

Image Contrast: Hot and Cold Spots

Reconstructed Image

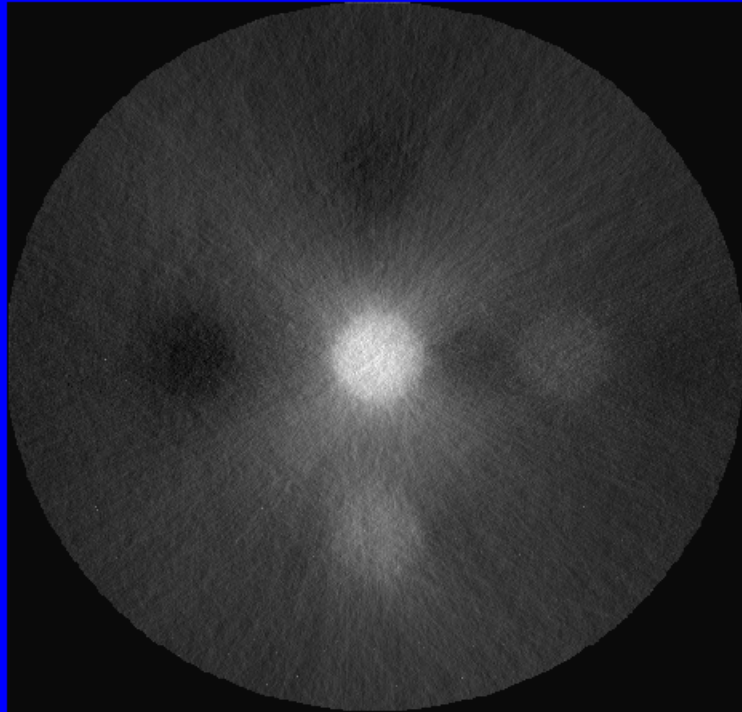
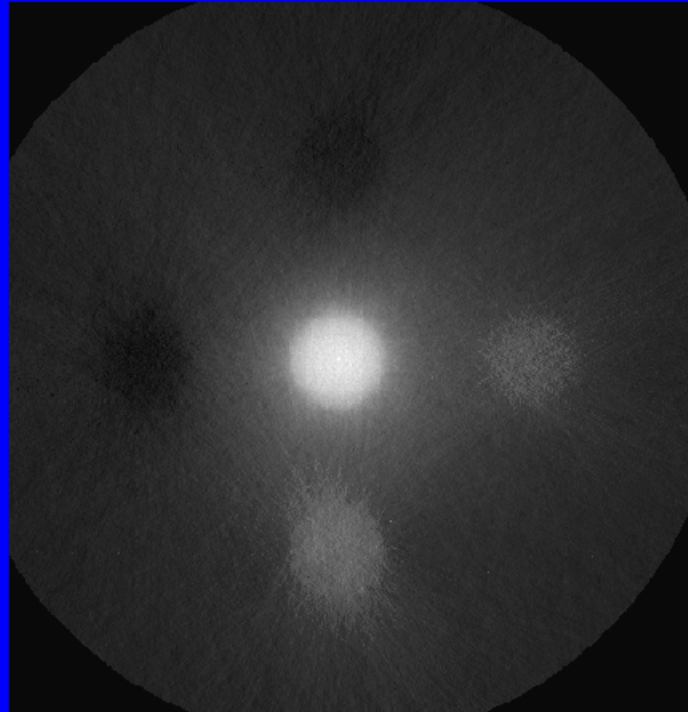


Image Deconvolved with PSF



Background: 100%;

Hot Spots: 130%, 120%, 110%;

Cold Spots: 90%, 80%

Results: Contrast

130% Max: 0.13, current: 0.076

80% Max: -0.18, current: -0.112

Summary of 511 keV Simulations

Negligible Doppler broadening uncertainty

Polarization did not affect sensitivity or resolution

Optimum design parameters:

Si detector thickness = 5 cm

Common diameter = 62.1 cm

NaI detector length = 78.4 cm

Si detector position = 0 cm

Angle = 12°

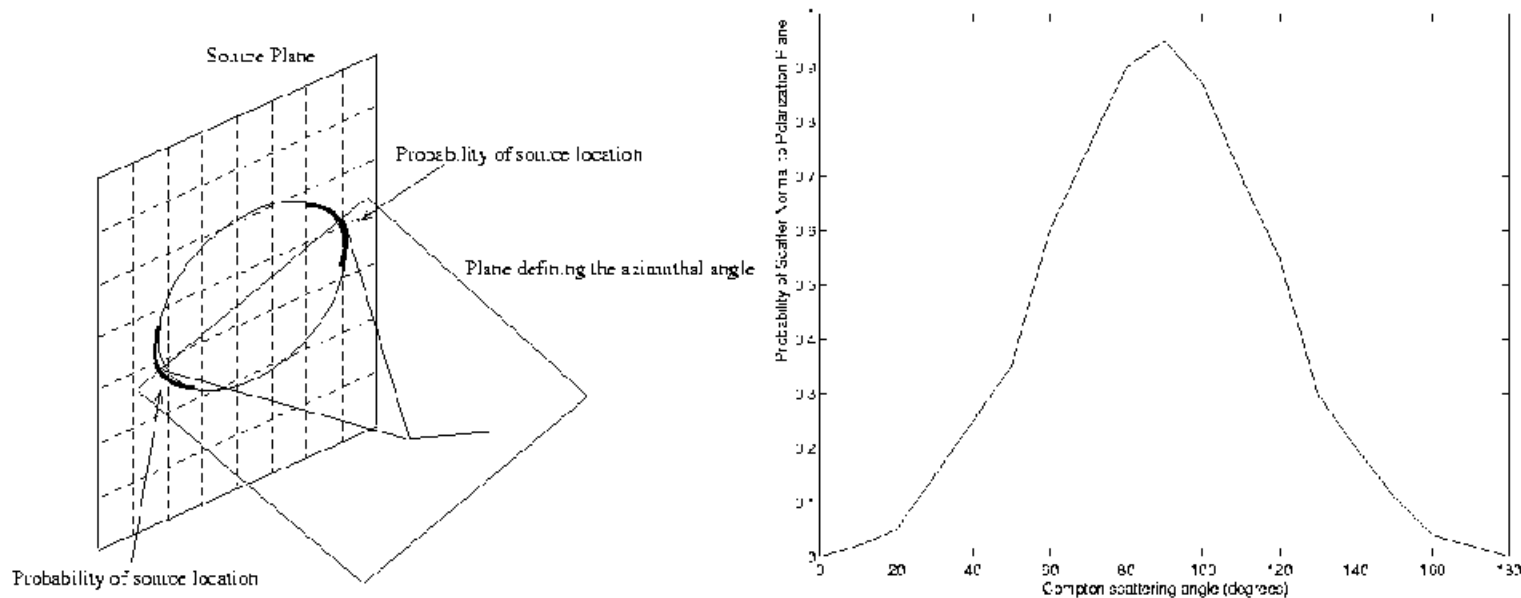
Sensitivity = 9731

Resolution = 0.731 cm

Polarization Based Reconstruction

Linearly polarized photons typically scatter into planes normal to the plane of polarization

Use information from multiple scatters to reduce uncertainty

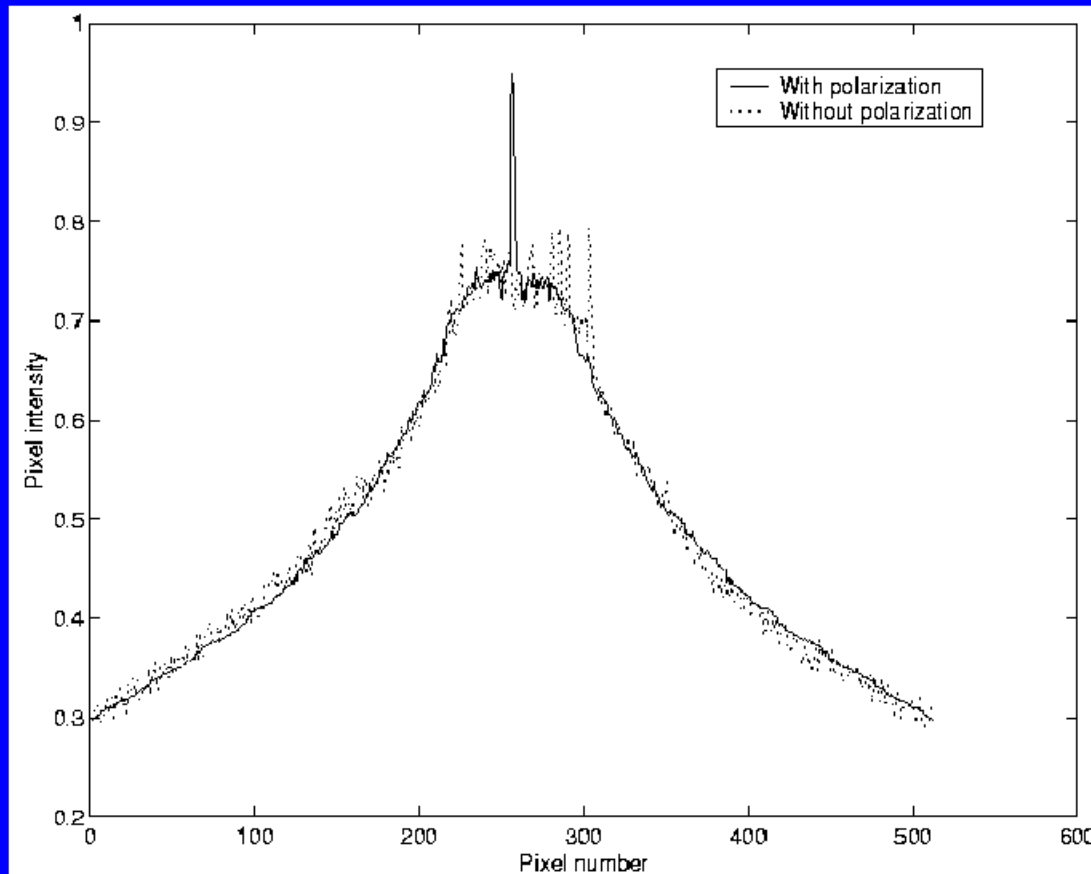


Reconstruction with Polarization

140 keV point source in sphere of 6 cm radius

Sphere contains water.

Data set: 10000 counts



Conclusions

Evaluated the design of a Compton camera at two energies

Modeled Doppler broadening and polarization

Computed sensitivities, resolutions and contrasts

Doppler broadening degrades performance for 140 keV

Less degradation for 511 keV

Polarization has no effect on sensitivity or resolution

Reconstructed with photon polarization information

Optimized the design

Suggestions for Future Work

Use different detector materials: Ge, Ar, Xe, BGO

Simulate different designs

Improved reconstruction algorithms:

Fully 3D reconstruction

Rigorous implementation of photon polarization

Test with voxelized phantoms